$$\boxed{\text{Solving the Schrodinger eq.}}$$

$$\begin{bmatrix} E_0 & -A \\ -A & E_0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = E_0 + A \begin{bmatrix} a \\ b \end{bmatrix}$$

$Q2 \Rightarrow \hat{H} = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix} \Rightarrow$ eigenvalues $E_1 = E_0 + A$

$E_2 = E_0 - A$

eigenvalues $E_0 a - Ab = E_0 a + A a$

$|E_1\rangle = \frac{1}{\sqrt{2}}(|+a\rangle - |-a\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$|E_2\rangle = \frac{1}{\sqrt{2}}(|+a\rangle + |-a\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

<u>I method</u>

$|\Psi(t)\rangle = C_+(t)|+a\rangle + C_-(t)|-a\rangle = \begin{pmatrix} C_+(t) \\ C_-(t) \end{pmatrix}$

$i\hbar \frac{d}{dt}\begin{pmatrix} C_+(t) \\ C_-(t) \end{pmatrix} = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix} \cdot \begin{pmatrix} C_+(t) \\ C_-(t) \end{pmatrix}$

$\begin{cases} i\hbar \dot{C}_x(t) = E_0 C_+(t) - A C_-(t) \\ i\hbar \dot{C}_-(t) = -A C_+(t) + E_0 C_-(t) \end{cases}$

$X = C_+ + C_- \Rightarrow C_+ = \frac{1}{2}(X+Y)$

$Y = C_+ - C_- \Rightarrow C_- = \frac{1}{2}(X-Y)$

$i\hbar(\dot{C}_x + \dot{C}_y) = E_0(C_+ + C_-) - A(C_+ + C_-)$

add subtract $\quad i\hbar \dot{X} = E_0 X - AX \rightarrow X(t) = X(0) \cdot e^{\frac{-i(E_0 - A)t}{\hbar}}$

$i\hbar \dot{Y} = (E_0 + A)Y \rightarrow Y(t) = Y(0) e^{\frac{-i(E_0+A)t}{\hbar}}$

$C_+(t) = \frac{1}{2}(X+Y) = \frac{1}{2} e^{-iE_0\frac{t}{\hbar}}\left(X(0)e^{iA\frac{t}{\hbar}} + Y(0)e^{-iA\frac{t}{\hbar}}\right)$

$C_-(t) = \frac{1}{2} e^{-iE_0\frac{t}{\hbar}}\left(X(0)e^{iA\frac{t}{\hbar}} - Y(0)e^{-iA\frac{t}{\hbar}}\right)$

suppose $|\Psi(0)\rangle = |-a\rangle$

$|-a\rangle = C_+(0)|+a\rangle + C_-(0)|-a\rangle$

$C_+(0) = 0, \quad C_-(0) = 1$

$X(0) = 1, \quad Y(0) = -1$

$C_+(t) = \frac{1}{2}e^{-iE_0\frac{t}{\hbar}}\left(e^{iA\frac{t}{\hbar}} - e^{-iA\frac{t}{\hbar}}\right) = i e^{-iE_0\frac{t}{\hbar}}\sin\left(A\frac{t}{\hbar}\right)$

$C_-(t) = e^{-iE_0 t/\hbar}\cos\left(A\frac{t}{\hbar}\right)$

$\boxed{i}$

II

$$c_1 \frac{1}{\sqrt{2}}|+a\rangle - \frac{c_1}{\sqrt{2}}|-a\rangle + \frac{c_2}{\sqrt{2}}|+a\rangle + \frac{c_2}{\sqrt{2}}|-a\rangle$$

$$|\psi(0)\rangle = \underset{c_1}{C_1}|E_1\rangle + \underset{c_2}{C_2}|E_2\rangle$$

$$\left(\frac{c_1+c_2}{\sqrt{2}}\right)|+a\rangle + \left(\frac{-c_1+c_2}{\sqrt{2}}\right)|-a\rangle \qquad \checkmark$$

$$|\psi(t)\rangle = C_1 e^{-i(E_0+A)\frac{t}{\hbar}}|E_1\rangle + C_2 e^{-i(E_0-A)\frac{t}{\hbar}}|E_2\rangle \qquad |\psi(0)\rangle = |-a\rangle$$

$$\langle E_1 | -a \rangle = C_1 \langle E_1|E_1\rangle + C_2 \langle E_1|E_2\rangle$$

$$|-a\rangle = C_1|E_1\rangle + C_2|E_2\rangle \implies C_1 = \langle E_1|-a\rangle = -\frac{1}{\sqrt{2}}$$

$$C_2 = \langle E_2|-a\rangle = \frac{1}{\sqrt{2}} \qquad\qquad E_1 = \frac{1}{\sqrt{2}}\left(|+a\rangle - |-a\rangle\right)$$

$$|\psi(t)\rangle = -\frac{1}{\sqrt{2}} e^{-iE_0 t/\hbar}\left(e^{-iAt/\hbar}|E_1\rangle - e^{iAt/\hbar}|E_2\rangle\right) \qquad \checkmark$$